

# A Quasi-Optical Mode Converter with a Bifocal Mirror

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**Abstract**—This paper presents the design procedure of a quasi-optical mode converter to transform any kind of  $TE_{mn}$  mode into a Gaussian wave beam and experimental results obtained in the particular case of the  $TE_{64}$  mode at 110 GHz. The quasi-optical system consists of a helical-cut launcher and a bifocal mirror, which is designed, using the techniques of geometric optics, to focus the radiation of the launcher into a Gaussian focal spot. Such a system was fabricated and tested for the transformation of the  $TE_{64}$  mode. The experimental results showed that about 80% of the power incident in the focal plane is focused into a small Gaussian-like spot of less than 20 mm diameter, while the 97% of the power is contained into the main  $TE_{64}$  lobe.

## I. INTRODUCTION

THE INCREASING demand for high-frequency, high-power microwave radiation for plasma heating in fusion experiments has pushed to the development of gyrotron oscillators operating in higher-volumic modes or even in whispering-gallery modes. Microwave generation in these modes imposes two serious problems: inefficient transmission and plasma heating due to wall losses and complex polarization. Therefore, the conversion of these modes into a low-order mode or into a Gaussian wave beam, which then will propagate in free-space or in a corrugated waveguide as the  $HE_{11}$  mode, is absolutely necessary. The use of waveguide components for this type of mode conversion has been proven complicated and inefficient. Although the  $TE_{64}$  is relatively low in the mode spectrum, the sequence of waveguide converters necessary for the transformation to  $HE_{11}$ , forms a long line of about 3.5 meters in length, and the conversion efficiency cannot surpass the moderate value of 75–80%.

An alternative technique for efficient transformation of whispering-gallery modes into a Gaussian wave beam, namely the Vlasov coupler, has been originally proposed in [1]. In general, it is a quasi-optical system involving a waveguide aperture of one of a variety of types, which serves as a launcher, and a series of reflectors, which serve to form the Gaussian wave beam and to steer it in the desired direction. Since the first results were highly encouraging, much effort has been given to the improvement of this type of quasi-optical converter [2]–[6], and several versions of it have been studied

and experimentally tested in case of azimuthally symmetric and whispering-gallery modes [7]–[12].

In this paper we study the performance of a Vlasov-type coupler operating in a higher-volumic mode, such as the  $TE_{64}$ , which has been selected as the working mode of the TH1505 gyrotron oscillator of Thomson Tubes Electroniques. The proposed quasi-optical system consists of a Vlasov-type helical launcher and a bifocal mirror, which is able to transform any kind of  $TE_{mn}$  mode into a Gaussian beam. Experimental results are presented in the particular case of the  $TE_{64}$  mode at 110 GHz. The general theory for the design of such a quasi-optical system is presented in Section II, while the experimental setup and results are presented in Section III.

## II. GENERAL THEORY

The longitudinal magnetic field component of a left-handed  $TE_{mn}$  mode, propagating in the positive  $z$ -direction in a cylindrical waveguide of circular cross-section, is given by:

$$H_z = H_0 J_m(k_c r) \exp[j(\omega t - kz + m\phi)] \quad (1)$$

where  $k_c = \nu_{mn}/r_w$  is the cutoff wavenumber,  $\nu_{mn}$  is the  $n$ th root of  $J'_m(x)$ ,  $r_w$  is the waveguide radius, and  $k = (k_f^2 - k_c^2)^{1/2}$  is the guided wavenumber, with  $k_f = \omega/c$  being the free-space wavenumber.

To simplify the analysis of the launcher and the bifocal mirror design, we use the methods of geometrical optics. Since the design will refer to a volumic mode, such as the  $TE_{64}$ , having relatively low field on the waveguide walls, it is expected that the approximations introduced by geometric optics will not significantly alter the actual diffraction picture. The discrepancy between geometric optics and diffraction becomes a serious problem for the designer when a real whispering-gallery mode has to be considered as the working mode [11]. Remarks on the validity of geometric optics techniques in the design of quasi-optical converters are reported in [13]. Thus, a left-handed rotating  $TE_{mn}$  mode can be decomposed into plane electromagnetic waves with wave vector

$$\mathbf{k}_f = k_c \cos \gamma \mathbf{e}_r - \frac{m}{r} \mathbf{e}_\phi + k \mathbf{e}_z \quad (2)$$

where  $\gamma = \arcsin(m/k_c r)$ . The transverse part of the wavevector  $\mathbf{k}_f$  is shown in Fig. 1(a). As these plane waves travel along the guide, they are reflected from the metallic walls at an angle  $\alpha = \arccos(m/\nu_{mn})$ , while their propagation direction is always tangent to a cylindrical surface, the caustic, with radius  $r_c = m/k_c$ , forming the “bounce angle”

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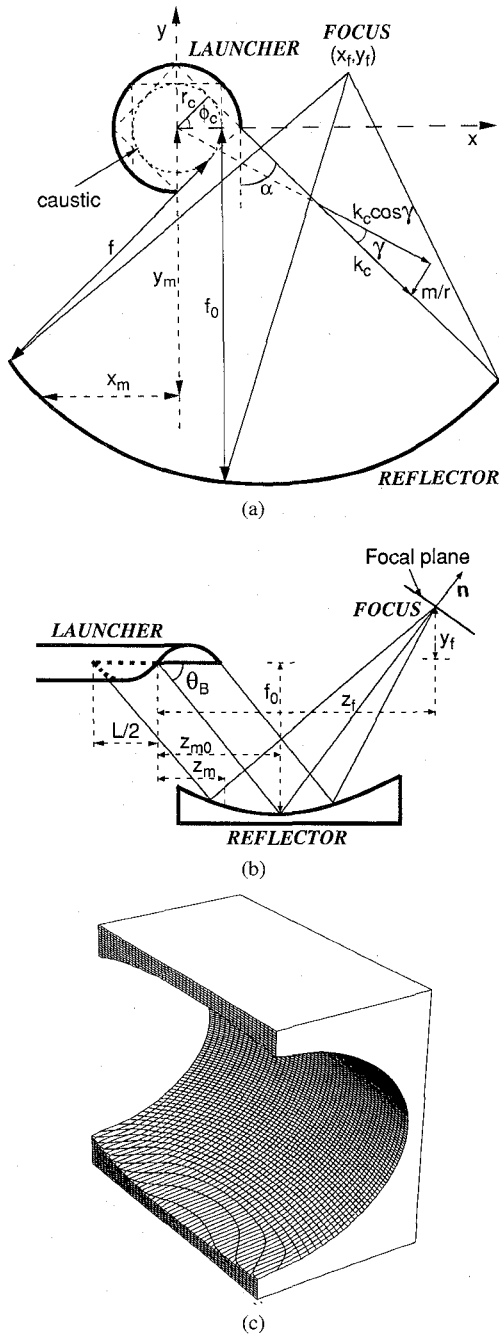


Fig. 1. (a) Cross-section of the coupler. (b) Side-view of the coupler. (c) Bifocal mirror.

$\theta_B = \arccos(k/k_f)$  with the  $z$ -axis of the guide. A picture of the ray paths is presented in Fig. 1.

The rays of the plane electromagnetic waves follow helical trajectories inside the waveguide. Thus the waveguide aperture, which serves to launch the waves to the reflector, is formed by a straight cut and a helical cut (see Fig. 1(b)), which actually follows the ray trajectories. The length of the straight cut of the launcher was first proposed to be the "bounce length" [1],

$$L_B = 2\pi r_w \tan \theta_B \quad (3a)$$

which is actually appropriate for real whispering-gallery modes  $TE_{m1}$  as the caustic radius of these modes is almost

equal to the waveguide radius. For higher volumic modes, the caustic radius is significantly smaller than the waveguide radius and the rays, which are tangent to the caustic, do not exactly skim along the waveguide wall. Equation (3a), which relates the length of the launcher's straight cut to the waveguide circumference and the bounce angle, is not valid for volumic modes and a modified length [7] is given by

$$L = L_B \frac{\sin \alpha}{\alpha} \quad (3b)$$

For real whispering-gallery modes, the reflection angle  $\alpha$  approaches zero and the difference between  $L$  and  $L_B$  is insignificant. For volumic modes, the length  $L$  becomes considerably shorter than the "bounce length"  $L_B$ . In particular, for the  $TE_{64}$  mode used in the experiment, the ratio between  $L$  and  $L_B$  is equal to 0.76. A more complicated expression for the length of the straight cut of the launcher, derived through energy considerations, is given in [12] and it is considered as the correct launcher length. The results obtained in that way agree with those obtained from (3b).

The condition of constant phase for the reflected rays at the focal point is then used for the design of the appropriate reflecting surface. In case of a left-handed rotating mode, this condition is fulfilled by the equation

$$-m\phi_c + kz_c + k_f(d_1 + d_2) = k_f d_0 = \text{constant} \quad (4)$$

where  $-m\phi_c + kz_c$  is the phase of the wave at a point  $(r_c, \phi_c, z_c)$  on the caustic surface,  $d_1$  and  $d_2$  are the optical lengths from that point of the caustic to the mirror and then to the focal point, and  $d_0$  is the total optical length, corresponding to an initial point on the caustic, which for convenience is chosen to be the middle of the launcher's straight cut  $z_c = 0$  at  $\phi_c = 0$ .

After some geometrical manipulations it is found that, at each cross-section  $z_m$  of the mirror, the coordinates  $x_m$  and  $y_m$  of the reflecting surface are given by

$$x_m = r_c \cos \phi_c + f \sin \phi_c \quad (5)$$

$$y_m = r_c \sin \phi_c + f \cos \phi_c \quad (6)$$

where  $f$  is the focal distance, that is the projection of the optical length  $d_1$  on the transverse plane (see Fig. 1(a)), given by the solution of the following quadratic equation

$$f^2 \cos^2 \theta_B - 2(\alpha_2 - \alpha_4 \sin \theta_B)f + (r_c - \alpha_1)^2 + \alpha_2^2 + \alpha_3^2 - \alpha_4^2 = 0 \quad (7)$$

where

$$\alpha_1 = x_f \cos \phi_c + y_f \sin \phi_c \quad (8a)$$

$$\alpha_2 = x_f \sin \phi_c - y_f \cos \phi_c \quad (8b)$$

$$\alpha_3 = z_f - z_m \quad (8c)$$

$$\alpha_4 = d_0 + \phi_c r_c \sin \theta_B - z_m \cos \theta_B \quad (8d)$$

and

$$d_0 = \frac{f_0}{\sin \theta_B} + [(x_f - r_c)^2 + (y_f + f_0)^2 + \left(z_f - \frac{f_0}{\tan \theta_B}\right)^2]^{1/2} \quad (8e)$$

In the above expressions  $x_f, y_f, z_f$  are the coordinates of the focal point, and  $d_0$  is the constant that appears in (4), given as a sum of two terms; the first is the optical length from the initial point  $\phi_c = 0, z_c = 0$  on the caustic to the reflector, with  $f_0$  being the corresponding focal distance, and the second term represents the length of reflected optical path from the mirror to the focal point.

### III. EXPERIMENTAL RESULTS

The above synthesis procedure has been applied to the design of a helical-cut launcher and a bifocal mirror to focus the radiation of the  $TE_{64}$  mode, propagating in a 27.8-mm-diameter waveguide at 110 GHz, into a small Gaussian spot.

The length of the launcher's straight cut, computed by (3b), is equal to 92.9 mm. The bifocal mirror, machined in solid Cu using a ball-end mill, is designed to focus the radiation to a focal point lying on the  $z$ -axis of the guide, at a distance of 335.3 mm from the middle of the launcher's straight cut, which is actually the "bounce distance." A picture of the reflector is given in Fig. 1(c). The reflector's cross-section is almost circular as the caustic radius (4.475 mm) is very small compared to the general dimensions of the system. In the axial direction, the shape of the reflector is similar to a parabola. Even for whispering-gallery modes, the shape of the reflector does not significantly change, because even in the extreme case where the caustic radius approaches the waveguide radius, it still remains much smaller than the dimensions of the system. For lower-order modes, the reflector's cross-section becomes more circular. The dimensions of the reflector are 250 mm  $\times$  120 mm  $\times$  260 mm. Machining precision is 0.01 mm, that is less than 0.5% of the free-space wavelength, and therefore losses due to surface imperfections are of the order of 0.2%. Displacements of the reflector along the axis of the output waveguide by 0.5 mm, or even more, did not significantly harm the quality of the output Gaussian beam. To ensure a good alignment, the reflector was mounted on a precision translator in such a way that transverse displacements were impossible. However, comparing the curvatures of the reflecting surface in the transverse and axial sections, we can estimate that the quasi-optical system is more sensitive to transverse than to axial misalignments.

The cold measurements were performed using the tunable, low power (1 W)  $TE_{10}$  mode, TH42210 carcinotron generator of Thomson Tubes Electroniques, and a series of mode converters to convert the  $TE_{10}$  first to  $TE_{01}$ , then to  $TE_{04}$ , and finally to  $TE_{64}$ . The mode purity is greater than 98%, with some power remaining in  $TE_{01}$  and  $TE_{04}$  modes. The output beam was measured with a WR8 diode detector and a 20-dB gain pyramidal receiving horn.

In Figs. 2 and 3 we present the  $E_\phi$  patterns of the  $TE_{64}$  mode and the radiation that comes out from the launcher. The angle of the main lobe in Fig. 3 is slightly smaller than the theoretical bounce angle ( $35.6^\circ$ ). This is because of a parallax effect introduced by the fact that the middle of the launcher's straight cut is at 125 mm from the rotation axis of the radiation bench.

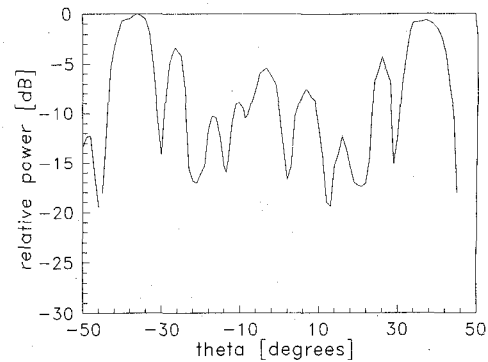


Fig. 2. Far field radiation pattern of the  $TE_{64}$  mode.

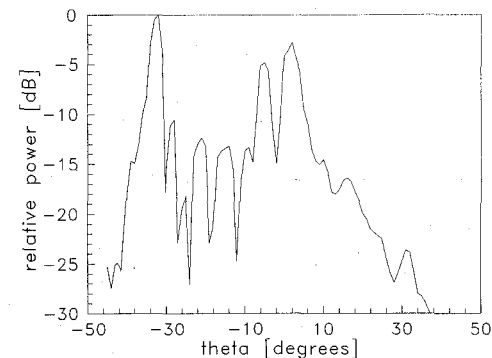


Fig. 3. Far field radiation pattern of the helical launcher.

Next, measurements were taken in the focal plane (at the focal point) keeping the receiving horn perpendicular to the axis connecting the center of the reflector and the focal point. The focal plane is defined by its normal vector  $\mathbf{n} = \sin \theta_B \mathbf{e}_y + \cos \theta_B \mathbf{e}_z$  (see Fig. 1(b)). The measured power densities of the  $E_\phi$  polarization in the focal plane, in the  $E$ - and  $H$ -planes, are shown in Fig. 4, along with the theoretical curves. The iso-power curves at the focal plane are shown in Fig. 5. As one can see from Fig. 4, the experimental curves fit well with the theoretical Gaussian curves, while the small sidelobes in the  $H$ -plane power density in Fig. 4(b) are probably due to spurious modes and especially due to the  $TE_{01}$ , which is radiated from the launcher almost unaffected by the presence of the reflector. These small sidelobes correspond to the two small spots in Fig. 5. From this figure one can see that the radiation is focused to a small spot of less than 20 mm diameter. The power density falls to the  $1/e$  of its on-axis value, in 9.6 mm in the horizontal  $x$ -direction, and in 8 mm in the vertical  $y$ -direction. The integration of the measured power density in the focal plane showed that nearly 80% of the power is focused in a 20-mm-diameter spot, while the 97% of the power is in the main  $TE_{64}$  lobe. Performing the cross-polarization measurements, we observed that the power in  $E_\theta$  was very low, at least 20 dB below that in  $E_\phi$ .

To see how much the beam fits to a Gaussian one we took measurements of the power density along the propagation axis of the beam in both the horizontal and vertical directions. Then, the beam's size was computed in these two planes, and it is shown in Fig. 6, along with the theoretical curves for the convergence and expansion of the fundamental Gaussian with

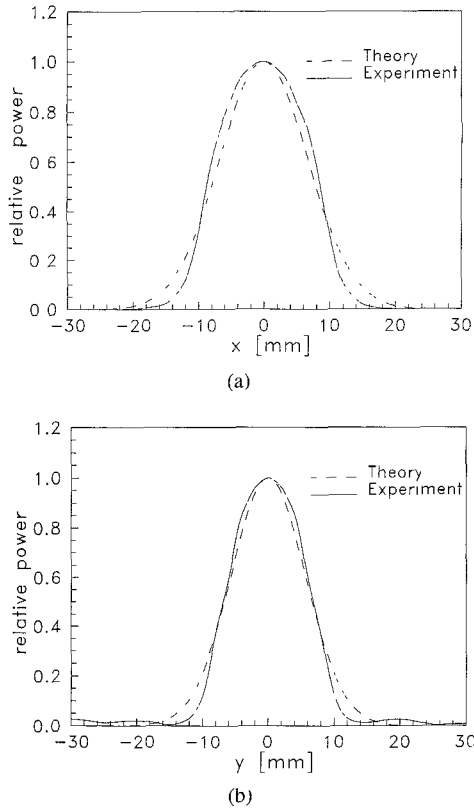


Fig. 4. Relative power density at the focal plane: (a)  $E$ -plane; (b)  $H$ -plane.

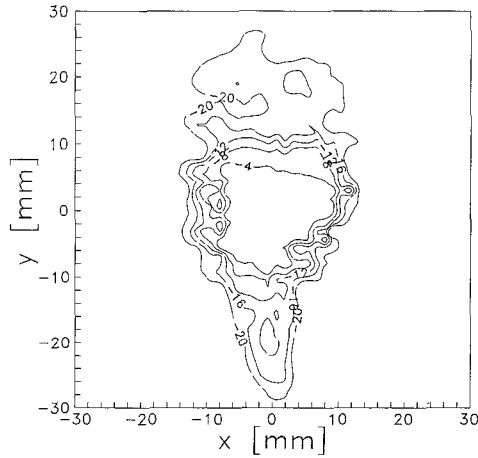


Fig. 5. Iso-power curves at the focal plane.

waist radii 9.6 mm in  $x$ - and 8 mm in  $y$ -direction. In this figure  $w_x$  and  $w_y$  are the beam radii in the  $x$  and  $y$  directions, in which the power density falls to the  $1/e$  of its on-axis value. Theoretically, at any distance  $\zeta$  from the waist position, and along the beam's propagation axis this  $1/e$  beam radius, for a Gaussian beam, is given by the formula

$$\frac{w(\zeta)}{w_0} = \sqrt{1 + \frac{\zeta^2}{q_0^2}} \quad (9)$$

where  $q_0 = \pi w_0^2 / \lambda$  is the Rayleigh range for a beam with waist radius  $w_0$ , and  $\lambda$  is the wavelength. As one can see in Fig. 6, the beam expands more rapidly than the fundamental Gaussian,

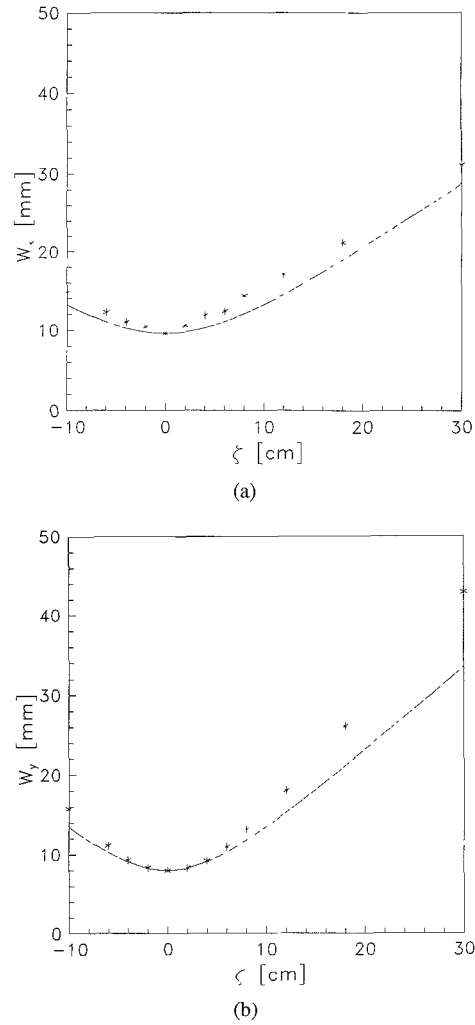


Fig. 6. Beam size along the propagation axis: (a) horizontal  $x$ -direction; (b) vertical  $y$ -direction.

especially in the vertical direction, while in the horizontal it expands with the same expansion angle as the fundamental Gaussian mode with waist 9.6 mm.

#### IV. CONCLUSION

A synthesis procedure, based on the geometric optics approximation, for the design of a quasi-optical converter to transform any kind of  $TE_{mn}$  mode into a Gaussian-like beam, as well as, the experimental results obtained in the particular case of the  $TE_{64}$  mode at 110 GHz, have been presented. The whole quasi-optical converter is quite compact and the transformation to a Gaussian beam is made in a length shorter than 40 cm. The experiment showed that the  $TE_{64}$  mode radiated by a helical-cut launcher is transformed, thanks to a bifocal mirror, into a linearly polarized Gaussian-like wave beam, which contains 97% of the power incident in the focal plane. The Gaussian beam was measured to be slightly elliptical with waist radii 9.6 mm and 8 mm in the  $x$ - and  $y$ -directions, respectively. Integration of the measured power density showed that at least 80% of the power incident to the focal plane is contained in a small spot of 20 mm diameter. Unfortunately, because of constraints imposed by the

measurement system, we were not able to measure the fraction of the power that was intercepted by the reflector, after the radiation leaves the helical launcher. Thus, we are not able to give a precise value of the diffraction losses, but an estimate is around 5%, due to the volumic mode and the oversized reflector used in the experiment. Therefore, one could say that the Gaussian beam formed by the quasi-optical converter contains about 92% of the power radiated by the launcher.

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